LOWER TOLERANCE LIMITS FOR SCREW WITHDRAWAL IN WOOD

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Abstract. In this study, the lower tolerance limits (LTLs) for screw withdrawal strength in wood were investigated. For this purpose, specimens were prepared from white oak and red oak wood (22.2 × 63.5 × 305 mm), a material widely used in furniture industry. Screw withdrawal tests were performed from transverse, radial, and tangential sections of wood specimens. Sample sizes for this study were determined by using modified Faulkenberry–Week methods. After considering normality, randomness, and homogeneity assumptions for tolerance analysis, LTLs were obtained from data sets in each sample group. According to screw withdrawal tests, ultimate tensile strength was 15.04 MPa, 17.93 MPa, and 16.77 MPa for red oak specimens; from each section, respectively. Likewise, those of white oak specimens were 16.36 MPa, 19.67 MPa, and 17.21 MPa, respectively. Results of LTLs for 0.99/0.99 confidence/proportional level were 8.69 MPa, 11.96 MPa, and 10.30 MPa for red oak specimens and 9.67 MPa, 11.14 MPa, and 11.58 MPa for white oak specimens from transverse, radial, and tangential sections of wood in screw withdrawal test, respectively. The study provides a systematic procedure to estimate design values for screws joints.

Keywords: Lower tolerance limits, screw withdrawal test, joint, reliability.

INTRODUCTION

Screws are widely used in furniture construction for the attachment of corner blocks to rails in chairs and tables, fastening tops to tables, cabinets and bases, attachment of shelves to end members, frames and trims to cabinets, and installing hardware (Feirer 1972). Wood screws were not only used for constructions in which joined members were loaded in pure tension or shear but also have been increasingly used to form moment resisting joints, such as those connecting a side rail to the back post of a chair—much like two-pin moment-resisting dowel joints (Eckelman 1971). Bending moment capacity of joints connected by screws (174 N.m) was performed and those of two-pin moment-resisting dowel joints (189 N.m) in the front-to-back cyclic load performance test (Uysal et al 2015).

Given such popularity of screws in the furniture industry, considerable information has been published, concerning the withdrawal and shear capacity of wood screws (Fairchild 1926; Corkrell 1933). Likewise, the Wood Handbook (1940) published expression for estimating the lateral and withdrawal capacities of wood screws, as a function of the specific gravity of the wood in which they were embedded—which have been updated over the years (Wood Handbook 2010). Similarly, Eckelman (1973, 1974, 1975, 1978) studied functionally related to withdrawal capacity of screws in wood and wood based composites. Ors et al (1998) demonstrated screw holding capacity of the oriental beech (Fagus orientalis L.), particleboard, medium density fiberboard, and werzalit wood-composite materials. Semple and Smith (2005) predicted internal bond strength of particleboard from screw withdrawal capacity as a function of density of the wood. Efe and Demirci (2005) investigated effect of cutting section to the holding strength of screw nut in specimens made

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Although numerous studies have been conducted to determine screw withdrawal strength in furniture construction, a critical unanswered question regarding this subject is what design value should be used for screws in furniture construction. An intermediate procedure that could be used to develop rational values for design purposes is the use of statistical lower tolerance limits (LTLs). Several studies were conducted to estimate LTLs of furniture joints. Eckelman et al (2016) demonstrated LTLs for T-shaped rectangular mortise and tenon joints constructed of red oak (*Quercus rubra* L.) and white oak (*Quercus alba*). Also, Eckelman et al (2017a, 2017b) studied the LTL approach to estimate equation-based rational design values for T-shaped rectangular mortise and tenon joints and L-shaped rectangular mortise and tenon joints, respectively. Uysal and Haviarova (2018) studied to estimate design values of dowel joints by using LTL method.

According to ANSI/BIFMA (2015), the overall goal in safety of furniture products is to reduce its injury rate in service. Moreover, Directive 2001/95/EC (European Parliament and the Council of the European Union, 2001) states that furniture product cannot cause any danger for its user because of its nature of application. When

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**Figure 1.** Schematic description of tolerance limits.

**Figure 2.** Wood sections.
reliability of product increases, its failure probability decreases. Joints are the weakest part of the furniture, so unreliable joints result in unreliable furniture (Smardzewski 2009). Reliability analysis estimates failure probability that predicts percentage of the material strength above its ultimate strength. In the case of reliability in furniture structure, its construction parameters must be determined by appropriate stress and strength distribution on structure under the imposed loading (Smardzewski 2009). Then, the joints can be easily designed with the given design values of these joints. In doing so, overall strength of furniture structure would increase as not exceeding internal stress on joints. At this point, the LTL value of joint strength denotes the previously sampled data to predict (λ) confidence limits on (βth) proportion of the future observation (Zaslavsky 2007). Figure 1 explains schematically how tolerance limits are constructed that a target population should be chosen and appropriate sample size should be chosen from this population to estimate observations which fall into some proportion of future population (β%) with some confidence level (λ%). Tolerance limits give an interval for future estimation rather than prediction interval and point estimation, which gives an estimation for single point for future observations.

In industry, one-sided tolerance limits are generally used in design acceptance sampling plans of quality controls. Whereas one-sided upper tolerance limits are used to determine acceptability of product characteristics, one-sided LTLs are used to determine reliability and safety of products (Ireson et al 1960). Material properties, such as strength, should be addressed to avoid structural failures. These properties also ensure that imposed load meets or exceeds the design value specified by lower percentiles of the quality characteristics. For such percentiles, one-sided LTLs are widely used and display more conservative estimates (Hu 2007).

Population parameters are practically unknown. They are estimated from sample statistics, so uncertainty due to sampling cannot be ignored (Rajagopalan 2004). In many fields of engineering design, it is important that proportion of population lies within specified limits, when using historical data set (Silva et al 2013). The data set in experiment covers a wide range of strength levels and gives predictions for overall material performance, but it is optimistic to determine design value without considering the variability of the data set (Saweeres et al 2005). At this point, it comes into prominence to determine what sample size should be used for the experiment. Young (2016) and Young et al (2016) studied R tolerance package to determine sample size for univariate normal data set by modifying the Faulkenberry–Weeks method (Faulkenberry and Weeks 1968).

The use of statistical LTL method for screw withdrawal capacity in wood enables a designer to quantify uncertainty. Specifically, in the design constructions, one can determine design the value of screws that might be expected to have lower withdrawal capacity than a specified withdrawal strength with a specified degree of confidence. Therefore, the initial questions that need to be answered in applying statistical LTLs to screw withdrawal capacity in wood enable a designer to quantify uncertainty. Specifically, in the design constructions, one can determine design the value of screws that might be expected to have lower withdrawal capacity than a specified withdrawal strength with a specified degree of confidence.

Table 1. Some physical and mechanical properties of wood used in specimen construction.

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Specific gravity</th>
<th>MOR (MPa)</th>
<th>MOE (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red oak</td>
<td>0.70</td>
<td>111.47</td>
<td>12,256.75</td>
</tr>
<tr>
<td>White oak</td>
<td>0.79</td>
<td>159.58</td>
<td>15,576.50</td>
</tr>
</tbody>
</table>

Figure 3. Screw configuration.
withdrawal capacities of wood are 1) how LTLs differ regarding change in confidence/proportion limits and 2) correspondingly, what sample size are needed to make reliable tolerance analysis.

MATERIALS AND METHODS

Material and Specimen Construction

In this study, red oak (Quercus rubra L.) and white oak (Quercus alba) wood, widely used in furniture structure, were used. Wood materials were obtained from local sawmill/lumber dealer located in northeast Indiana. Defect-free 22.225-mm-thick, 63.5-mm-wide, and 304.8-mm-long specimens were prepared for screw withdrawal test from transverse, radial, and tangential sections of wood (Fig 2). Some physical and mechanical properties of wood used in specimen construction are given in Table 1. Specific gravity of the wood was calculated according to Eckelman (1997) and MOR and MOE of wood were calculated according to ASTM D 143–94 standard.

Dimensions of stainless steel, button head, and coarse thread screw used in the study are shown in Fig 3. For specimens of screw withdrawal test from transverse and radial sections of wood, a 3-mm-diameter and 25.4-mm-long pilot hole was drilled; those of tangential section were 22.225 mm. Depth of penetration of screws in the transverse and radial section of wood for withdrawal tests was 25.4 mm. In the screw
withdrawal test from tangential section of wood, screws were embedded until a distance of 3.175 mm between the screwhead and bottom face of the specimen was reached (Fig 4) (Eckelman and Cassens 1984). After insertion of the screws, the specimens were stored in a conditioning chamber to maintain 7% MC (Erdil et al 2002). During the experiment, the MC of specimens was held approximately at a level of 7%.

Test Procedure

All tests were conducted on an MTS universal test machine with 220 kN load capacity. Screw withdrawal tests from transverse, radial, and tangential section of wood were conducted using the test setup shown in Fig 5 at a cross head loading rate of 12.7 mm/min (Eckelman and Cassens 1984). Loading was continued until a nonrecoverable drop-off in load occurred. Ultimate withdrawal load (N), \( F \), read from load-deformation curve and tensile stress for screw withdrawal of wood, was calculated using Eqs 1 and 2 (Efe and Demirci 2005):

\[
\sigma_T = \frac{F}{A},
\]

\[
A = \pi \times D \times L,
\]

where \( \sigma_T \) is the tensile stress, \( A \) is the joint area (mm\(^2\)), \( D \) is major screw diameter (mm), and \( L \) is the screw effective length (mm).

Determination of LTLs

To conduct a reliable tolerance analysis, it must be determined whether the data set is normally distributed or not. In the case of a normally

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Wood section</th>
<th>Mean (MPa)</th>
<th>SD (MPa)</th>
<th>Variance (MPa)</th>
<th>CoV (%)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red Oak</td>
<td>Transverse</td>
<td>15.51</td>
<td>2.63</td>
<td>6.94</td>
<td>17</td>
<td>0.8652</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>17.63</td>
<td>2.53</td>
<td>6.42</td>
<td>14</td>
<td>0.9254</td>
</tr>
<tr>
<td></td>
<td>Tangential</td>
<td>17.11</td>
<td>2.39</td>
<td>5.72</td>
<td>14</td>
<td>0.4504</td>
</tr>
<tr>
<td>White Oak</td>
<td>Transverse</td>
<td>17.29</td>
<td>2.47</td>
<td>6.09</td>
<td>14</td>
<td>0.3396</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>19.03</td>
<td>2.89</td>
<td>8.37</td>
<td>15</td>
<td>0.4088</td>
</tr>
<tr>
<td></td>
<td>Tangential</td>
<td>17.39</td>
<td>2.44</td>
<td>5.96</td>
<td>14</td>
<td>0.0991</td>
</tr>
</tbody>
</table>
distributed data set, one-sided LTLs are calculated by using the following expression:

\[
LTL = \bar{X} - (k_{n, \lambda, \beta} \times s),
\]

where \(\bar{X}\) refers to the average capacity of the test results, \(s\) refers to the standard deviation of the results, and \(k\) is a tolerance factor depending on the sample size and confidence/proportion limits (Natrella 1963). Although \(z\)-statistic is used to determine the \(k\)-tolerance factor for tolerance limits, there is an error when the sample size is less than 30. Therefore, Link (1985) proposed an equation for the \(k\)-tolerance factor by using the Guttman (1970) theorem and noncentral \(t\)-distribution approximated by standard normal distribution. For small sample sizes, assumption for normal distribution is poor because of underestimation of sample size. Hence, \(t\)-distribution must be used when the sample size is less than 30. In this case, the \(k\)-tolerance factor is calculated by using the following formula (Young et al 2016):

\[
k_{n, \lambda, \beta} = \frac{1}{\sqrt{n}} \times t_{n-1, \lambda, \delta},
\]

where \(\delta\) is the noncentrality parameter and \(z_{\beta}\) is the \(z\)-statistic for proportion limit.

If the data set is not normally distributed, logarithmic normalizing transformation was performed. Then, if logarithmic data are normally distributed, LTLs are calculated for transformed data and results are inverted. If logarithmic transformation fails, alternative distribution, such as the Weibull distribution, is used and LTLs are calculated based on those distributions. If any of the aforementioned methods works, LTLs are
calculated as nonparametric tolerance analysis by using binomial probability:

\[ P(X_i < \xi) = \binom{n}{x} \times p^x \times q^{n-x}, \]  

where \( X_i \) refers to the values below the LTL, \( \xi \) is the LTL value, \( P \) is the significance level (\( \alpha = 1 - \lambda \)), \( n \) is the sample size, \( p \) is the proportion level (\( \beta \)), and \( q \) is (\( 1 - \beta \)).

Figure 7. Ultimate screw withdrawal strength from transverse, radial, and tangential sections of wood.

<table>
<thead>
<tr>
<th>Wood specimen</th>
<th>Wood section</th>
<th>Mean (MPa)</th>
<th>SD (MPa)</th>
<th>Minimum value (MPa)</th>
<th>Maximum value (MPa)</th>
<th>Range of minimum and maximum values %</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red oak</td>
<td>Transverse</td>
<td>15.04</td>
<td>2.38</td>
<td>9.36</td>
<td>21.61</td>
<td>57</td>
<td>0.3553</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>17.93</td>
<td>2.68</td>
<td>12.80</td>
<td>26.23</td>
<td>51</td>
<td>0.0041</td>
</tr>
<tr>
<td></td>
<td>Tangential</td>
<td>16.77</td>
<td>2.43</td>
<td>11.48</td>
<td>23.43</td>
<td>51</td>
<td>0.1920</td>
</tr>
<tr>
<td>White oak</td>
<td>Transverse</td>
<td>16.36</td>
<td>2.52</td>
<td>8.54</td>
<td>22.82</td>
<td>63</td>
<td>0.2683</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>19.67</td>
<td>3.21</td>
<td>12.73</td>
<td>30.06</td>
<td>58</td>
<td>0.3362</td>
</tr>
<tr>
<td></td>
<td>Tangential</td>
<td>17.21</td>
<td>2.52</td>
<td>10.83</td>
<td>25.39</td>
<td>57</td>
<td>0.0053</td>
</tr>
</tbody>
</table>
R tolerance package is also provided to estimate tolerance intervals for normally distributed data sets, Weibull distribution, and nonparametric tolerance analysis (Young 2010).

RESULTS AND DISCUSSION

Sample Sizes for Tolerance Analysis

According to the central limit theorem, 30 specimens, as shown in Fig 4, were used to determine minimum sample size requirements for screw withdrawal tests as the reference data. Sample statistics for reference data are given in Table 2. To apply the modified Faulkenberry–Weeks method, the data set must be normally distributed. Therefore, the Shapiro–Wilks normality test was performed to determine the normality of the data sets. Results show that all data sets are normally distributed because the p-values are greater than 0.05 for each sample group.

Minimum sample sizes were determined for 0.90/0.90, 0.90/0.95, 0.90/0.99, 0.95/0.90, 0.95/0.95, 0.95/0.99, 0.99/0.90, 0.99/0.95, and 0.99/0.99 confidence/proportion levels. Specification limits were chosen as means of reference data minus 3σ (X̄−3σ) because of the classic rule of thumb used for setting specification limits (Young et al 2016). Results are shown in Fig 6. In all confidence/proportion levels, the sample sizes were same for each sample group but it differed for the 0.99/0.99 confidence/proportional level. These variations may occur because of the coverage of future sampling. According to these results, screw withdrawal tests of red oak specimens from the tangential section of wood required the largest sample size to make tolerance analysis at 0.99/0.99 confidence/proportion level. For this purpose, 220 specimens were made for each sample group (1320 specimens in total) to ensure homogeneity in sample sizes.
LTLs for Screw Withdrawal of Wood from End-, Edge-, and Face-Grain

Results of ultimate screw withdrawal strength from transverse, radial, and tangential section of red oak and white oak wood are shown in Fig 7 and Table 3. The range of variability could be clearly observed in the data set, so the use of deterministic approach would be more optimistic to determine the design value by using only sample statistics. Screw withdrawal strength in white oak wood from all sections of wood were higher than those of red oak because density of white oak (0.68 g/cm³) is higher than the density of red oak (0.63 g/cm³) (Wood Handbook 2010). For the sample group of red oak, average screw withdrawal strengths from transverse, radial, and tangential sections of wood were 15.04 MPa with a standard deviation of 2.38 MPa, 17.93 MPa with a standard deviation of 2.68 MPa, and 16.77 MPa with a standard deviation of 2.43 MPa, respectively. The average screw withdrawal strengths from transverse, radial, and tangential sections of white oak wood were 16.36 MPa with a standard deviation of 2.52 MPa, 19.67 MPa with a standard deviation of 3.21 MPa, and 17.21 MPa with a standard deviation of 2.52 MPa, respectively. The highest withdrawal strength in both red oak and white oak wood was obtained from the radial section of wood because in this...
case, the wood fiber orientation is perpendicular to screws, whereas those of transverse section of wood have the lowest withdrawal strength in wood. Therefore, screw withdrawal from radial and tangential section of wood may have more effective endurance (Efe and Demirci 2005).

P-values are also presented in Table 3 for normality of data sets using the Shapiro–Wilks test. Sample groups were normally distributed because the p-values were greater than 0.05, with exceptions of sample groups of withdrawal tests from radial section of red oak wood specimens (p-value = 0.0041) and tangential section of white oak wood specimens (p-value = 0.0053). Besides, normal quantile (Q-Q) plots for sample groups are shown in Fig 8. Logarithmic normalizing transformation was used to calculate LTL values for these non-normal data sets and then logarithmic LTL values were inverted to LTL values. After logarithmic normalizing transformation was applied, the p-values were 0.2718 and 0.8485 for radial section of red oak wood and tangential section of white oak wood specimens, respectively. Therefore, Eq 3 can be used for the transformed data set as well.

Results of LTL values for screw withdrawal strength from transverse, radial, and tangential sections of red oak and white oak wood are given in Fig 9. At the same sample size, the LTL values decreased, whereas confidence/proportion levels were ascending because the k-tolerance factor increased with the confidence proportional level. Moreover, an increase in the proportional level had a larger effect than an increase in the confidence level. The difference between LTL values at 0.95/0.95 level (10.68 MPa) and 0.99/0.95 level (10.48 MPa) for transverse section of red oak wood was 1.87%. On the other hand, the difference between 0.95/0.95 level (10.68 MPa) and 0.95/0.99 level (8.94 MPa) was 16.29%.

LTL values at different sample sizes are shown in Fig 10. Moreover, corresponding sample statistics, results of the Shapiro–Wilks normality test, and difference between LTL values of sample size of 220 and various sample sizes are tabulated in Table 4. The p-values were greater than 0.05 for all sample groups, so Eq 3 was used for all LTL calculations. With all sample sizes, LTL values show differences due to mean, standard deviation, and the k-tolerance factor. However, such difference is growing narrower when increasing the sample sizes because k-tolerance factors were reasonably high for narrower sample sizes. The choice of sample size changes depending on how reliable joint would one like to produce. In the case of heavy loading on furniture, high confidence/proportion level should be present to reduce failure probability by increasing the reliability of joints. If the designer chooses the 0.99/0.99 level to design a joint (with corresponding screw size, screw penetration, and number of screws), then the failure probability would be 1% with 99% confidence level. On the other hand, a lower confidence/proportion level could be chosen when high reliability of furniture structure is not a required feature. For example, the lower confidence/proportion level LTLs values for screw withdrawal strength could be used in the case of joining top shelves of a bookcase to its sides.

CONCLUSION

In this study, the LTLs for screw withdrawal strength from transverse, radial, and tangential sections of red oak and white oak wood were determined. Results of the study illustrated that the use of LTL method provides a systematic
approach to estimate the design value of screw withdrawal in wood.

In tolerance analysis and determination of sample size, normality assumptions are vital to make a reliable analysis. Modified Faulkenberry–Weeks method, proposed by Young et al (2016), was used to determine the sample size. At different confidence/proportion levels, the sample sizes vary. To make a reliable tolerance analysis at 0.99/0.99 confidence/proportion level for all sample groups, 220 specimens for each sample group (1320 specimens in total) were analyzed to determine ultimate screw withdrawal strength in wood. The Shapiro–Wilks normality test was conducted to determine whether data were normally distributed or not. If yes, expression of \( \bar{X} - (k \times s) \) was used directly to calculate LTLs at different confidence/proportional levels. Otherwise, logarithmic normalizing transformation was performed for non-normal data sets. The same LTLs expression was used to calculate LTLs for transformed data. Then, they were inverted.

Change in the confidence level for tolerance analysis has more effect than those of the proportion level because proportion level pertains the future population, whereas confidence level pertains the sample. Results also proved this phenomenon.

Preference of confidence/proportion levels changes depending on how reliable joint would be used in furniture construction. For high-risk joints, LTL values at 0.99/0.99 confidence/proportion levels are recommended for joint design (screw size, penetration of screw, and number of screws in joint).

To produce reliable joints, it is recommended for screw withdrawal strength in wood not to exceed the given LTL values at the preferred confidence/proportion level. Thus, reliable joint design should result in reliable furniture construction.

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