Minimizing the Weight of a Flooring Strip: a Shape Optimization Approach

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Abstract

For economical and environmental reasons we are interested in diminishing the weight of flooring strips. One way to achieve this is to use the number and shape of grooves underneath the strip. Using warping as a comparative tool, we could analyze the merit of a finite number of designs. However, with this approach we cannot guarantee that the result is the most favourable. The search for the ``best design" leads to design optimization: minimizing the weight by acting upon a part of is shape taking into account its warping or stiffness.

We present an optimization strategy adapted to the calculation of the optimal design subjected to arbitrary mechanical and geometrical conditions (for the flooring strip we have a condition on the thickness of the wear layer). This approach is not limited to flooring strip and can be used in any situation where a linear hygromechanical model is relevant. This strategy is composed of two steps: global optimization with respect to admissible variations of the shape (or design) followed by a post-processing phase taking into account various other mechanical and possibly geometrical conditions imposed on the strip.

Keywords: design optimization, finite element method, weight, stiffness, moisture content, linear elasticity

Introduction

For economical and environmental reasons we are interested in diminishing the weight of parts made of wood or wood fibre composites, in particular flooring strip. A reduction in weight will lead to a better use of the resources and improved shipping. The analysis of various design of parts made of any material driven by hygrometric (and possibly thermal) conditions can be a difficult and time-consuming task. In that context the development of a simulation tool allowing a reduction in the research cycle is interesting.

Variations of hygrometric conditions can induce undesirable hygromechanical deformations. For appearance products such as parquet, even a small deformation of the material is unacceptable. Adsorption and desorption of water vapour may induce cupping, and consequently decrease product value. In this context, it is of primary importance to reduce warping due to variations of the thermo-hygrometric conditions. Using the warping as a comparative tool, we could analyze the merit of a finite (discrete) number of designs relative to some hygrometric variations. But we are interested in avoiding the subjectivity of the discrete design optimization. With the objective of developing a systematic and unbiased way of lowering the weight while maintaining acceptable dimensional stability comes the incentive to optimize their design.

The hygromechanical analysis follows Deteix (2008). The numerical model relies on the three-dimensional finite element approximation of the solution of the hygromechanical model. For the shape optimization part, first we devised a strategy (loosely inspired by Mins (1999)) allowing us to place this structural optimization problem with transient loading in the context of classical shape optimization (Bendsøe and Sigmund (2002), Delfour and Zolésio (2001)). The numerical approach is based on the SIMP method (Solid Isotropic Microstructure with Penalization) as presented in Bendsøe (2002). We used the heuristic approach of Sigmund (1997) for the filtering of the checkerboard effect (numerical artefact produced by certain choice of degree of interpolation of the displacements) and making the design independent of the finite element grid.

Hygromechanical Model

We want to describe the mechanical behaviour of a wood or wood composite part $\Omega \subset \mathbb{R}^3$, subjected to mass transfer (moisture movement). We consider that we have isothermal conditions and a variation of the hygrometric conditions. The mass transfer occurs by free convection from the surfaces. The transient moisture movement is described by the three-dimensional moisture conservation equation.

$$\frac{d_b}{100}\frac{\partial M}{\partial t} + \vec{\nabla}\cdot\left(-K_M\,\vec{\nabla}\,M\right) = 0 \text{ with } K_M = \frac{D\,d_b}{100} \tag{1}$$

where K_M: tensor of effective water conductivity (kg m^{-1} s^{-1} $\%^{-1}$); D: tensor of effective moisture diffusion (m^2 s^{-1}) ; d_b : basal density (kg m^{-3}) and M: moisture content (%).

The wood is orthotropic and elastic (satisfying Hooke's law). No mechano-sorptive effects are taken into account (Blanchet *et al*. 2003). Since we consider that the material is conditioned, the flooring strip is initially free of stress. The governing equation for the description of the mechanical aspect of the problem is the three-dimensional (incremental quasi-static state) equation of equilibrium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \qquad \sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \beta_{kl}\Delta M) \tag{2}$$

where body forces are assumed to be negligible. σ_{ij} are the normal and shear stress components, expressed in a rectangular coordinate system, C_{ijkl}: stiffness tensor; ε_{kl}: strain tensor; β_{kl}: moisture shrinkage/swelling coefficients ($\%^{-1}$); ΔM: moisture content change between two time steps (%). The stiffness tensor is constant (in time, no ageing) and we take into account hysteresis in adsorption/desorption for the shrinking/swelling coefficients (Goulet-Fortin 1975). The strains are related to the displacements, u_1, u_2 and u_3 measured along the x_1, x_2 and x_3 directions, respectively.

$$\varepsilon_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

Shape Optimization under Transient Conditions

The mechanical behaviour of a flooring strip (or of any wood or wood composite part) is transient. The search of an optimal design (we will use, without distinction, the terms shape, design or geometry to describe the complete definition of the geometry of the part under study.) should take into account this transitory nature. This implies to set up an optimization process to establish the most favourable design (shape) in relation to weight reduction and suitable stiffness over a fixed period of time [0, t_{max}]. We propose an approach, inspired by Min et al (1999) which is mathematically sound and easy to implement. This approach, called Transient Shape Optimization Analysis (TSOA) consists in determining the optimal shape at a fixed time (critical time) followed by the verification of a set of mechanical criteria on the design for the whole period. When the criteria are satisfied, we consider our design optimal and in case of failure the process is restarted using the current shape and the time at which failure of the criteria occurred as the new critical time (Figure 1)

Figure 1 The TSOA paradigm, an optimization process for transient behaviour/cost function.

Shape Optimization of the Unsteady Strain Energy

Recall that we have denoted by Ω the domain of $\mathbb{R}^3$ occupied by the part. This domain will be included in maximal domain Ω^{Max} and composed of two components Ω_F and Ω_G

$$\Omega = \Omega_F \bigcup \Omega_G \quad \Omega_F \bigcap \Omega_G = \varnothing \quad \varnothing \neq \Omega_G \subseteq \Omega_G^{Max} \tag{4}$$

where Ω_F is a fixed component (possibly empty) and Ω_G is the zone where variation of the geometry is permitted and contained in a larger and fixed research region Ω_G^{Max} (Figure 2).

Figure 2 Two-dimensional example of the geometrical decomposition. On the left the maximal domain (known as the hold-all). On the right, an example of possible design included in the hold-all with a variation of the geometry contained in the modifiable zone.

For Ω_F and Ω_G^{Max} given, we define the set of admissible shapes Π_{ad} as the set of domains satisfying Equation (4).

$$\Omega_F, \Omega_G^{Max} \subset \mathbb{R}^3, \Omega_G^{Max} \neq \varnothing, \quad \Pi_{ad} = \left\{ \Theta \subset \mathbb{R}^3 \middle| \Theta \text{ satisfy (5)} \right\} \tag{5}$$

$$\forall \Omega \in \Pi_{ad} \quad \Omega \subseteq \Omega_F \bigcup \Omega_G^{Max} = \Omega^{Max} \tag{6}$$

We are looking for an admissible optimal shape, the optimization process is restricted to Π_{ad}. The weight is taken at moisture content M_{ref} and is a function of the basal density (FPL 1999):

$$W\ (\Omega)\ =\ 1000\int_{\Omega}\frac{0.3(100+M_{ref})d_b}{30000-0.265(30-M_{ref})d_b}d\Omega \tag{7}$$

We use the strain energy $S(t,\Omega)$ as a global indication of warping. The displacements are fixed only on a finite number of points and we have:

$$S(t,\Omega)=\int_{\Omega}\left(C_{ijkl}\int_0^t\beta_{ij}\frac{dM}{dt}ds\right)\varepsilon_{kl}(u)d\Omega+\int_{\Gamma_N}f_N\cdot u\,d\Gamma \tag{8}$$

where f_N is the load applied on Γ_N, a part of the boundary.

We chose to minimize the displacements (via the compliance (Eq. (8))) while imposing a restriction on the weight. This approach is classical, it is the usual approach presented in the literature (Delfour (2001), Bendsøe (2002)). A priori, it is easier to fix a goal on the reduction of weight and to try to achieve it with a part as rigid as possible. We will maximize the stiffness for an imposed minimal reduction of weight (at least $100*\alpha$ % of the weight of Ω_G^{Max}).

$$\min_{\Omega\in\Pi_{ad}} S(t,\Omega)$$

such that

$$W(\Omega_G)\leq(1-\alpha)W(\Omega_G^{Max})\quad 0<\alpha<1$$

with *M* and *u* solution of (Eqs. (1) and (2)) on $[0,t_{\max}]$ with appropriate initial and boundary conditions.

In order to solve this problem using the TSOA illustrated in (Figure 1), we need to elaborate on the classical (i.e. steady objective function and state variable) shape optimization.

Static Shape or Topological Optimization

Shape optimization in its most general setting should consist of a determination for every point in space whether there is material in that point or not. As the theory related to shape optimization evolved, a clear distinction between shape (perturbations of the boundaries of an established shape) and topological (determination of the topology (holes, disjoint components) of the shape) optimization emerged. These techniques are generally used at different stages of optimization and performed separately. Topological optimization is usually considered as a tool for finding efficient design concepts at the early design stage, whereas shape optimization is viewed as a tool for detailed design at a later stage. In either case, the theory is based on a finite element (FE) mesh used to discretize the design domain.

In topological optimization the topology of the structure is not fixed a priori (it is possible to generate/eliminate holes and disjoint components). Alternatively, for a finite element discretization, every element is a potential void or structural member. Topological problems formulated this way are inherently discrete optimization problems but there are various ways of solving them without the use of discrete optimization algorithms. One of the most effective method is the SIMP approach (Solid Isotropic Material with Penalization) (Bendsøe 2002). Here, material properties are assumed constant within each element of the FE mesh and the variables are the element volume fraction of solid. The material properties are modeled as:

$$\theta(\rho) = \rho^p \theta^0 \tag{9}$$

where θ is a material property, θ^0 the solid material property, ρ the volume fraction of solid and p the power of the penalizing. This approach eliminate the use of complex rule of mixture for the intermediate values of volume fraction and in fact, for reasonable values of p, gives solution with very little, if any, intermediate values.

As a first approach in shape optimization for the flooring strip, we will favour the topological optimization. The SIMP method will be used since it can be applied to problems with multiple constraints, multiple physics and multiple materials allowing us for a more general problem.

The two main numerical difficulties of the SIMP are the checkerboard effect (regions with alternating solid and void elements) and the mesh dependency (qualitatively different design for refined FE mesh). A review of these problems and their treatment can be found in (Bendsøe and Sigmund 2002, Bruns 2005). Here, a mesh-independency algorithm that both eliminate the checkerboard problem and the mesh-dependency problem is applied. The method works by substituting the element-wise derivatives with respect to the volume fraction (sensitivities) with a weighted average of the sensitivities of their neighbours within a given radius.

Topological Optimization for a Flooring Strip

We consider a flooring strip made of sugar maple as shown in Figure 3. This basic flooring strip, geometrically composed of three rectangular prisms, will be considered our hold-all Ω^{Max} (we are looking for shapes contained into this basic shape). The top surface is 107.9 mm-wide by 609 mm-long. The construction considered is a 7.2 mm-thick surface layer, a 6.4 mm-thick core layer and a 5.1 mm thick backing layer for a total thickness of 18.7 mm and with tongue and groove of 5.5 mm deep.

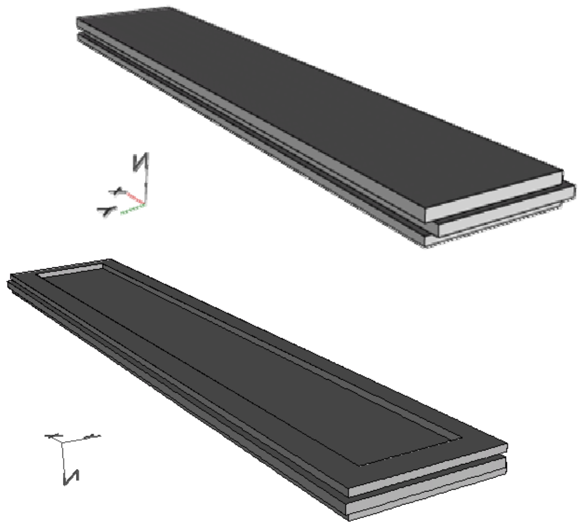

Figure 3 The basic flooring strip used as hold-all for the model. On the right the flooring strip view upside down, view of Ω_G^{Max} (here without material)

We chose the final time to be of 42 days: we expose a flooring strip to an ambient relative humidity of 80% for a period of 42 consecutive days. From a finite element analysis for the hold-all we conclude that the first critical time of the TSOA should be at 18.5 days (which coincides approximately to the time where we observe extreme values of displacements).

Using the fact that the stiffness tensor is constant in time and that no load is applied (freestanding condition) on the flooring strip we have

$$t_{opt} = 18.5 \text{ days} \quad s_{ij}^{opt} = \int_0^{t_{opt}} \beta_{ij} \frac{dM}{dt} ds \quad S(t_{opt}, \Omega) = S_{opt}(\Omega) = \int_\Omega C_{ijkl}\, s_{ij}^{opt}\, \varepsilon_{kl}(u) d\Omega \quad \forall \Omega \in \Pi_{ad} \tag{10}$$

$$-\nabla \cdot (C_{ijkl}(\varepsilon_{kl}(u) - s_{kl}^{opt}) = 0 \tag{11}$$

where M is the solution to (Eq. (1)) over the 42 days on the hold-all Ω^{Max} and u solution of (Eq. (11)) with appropriate initial and boundary conditions for both. Assuming a constant basal density and since we chose to measure the weight at the initial moisture content, which is constant through out the hold-all, the weight constraint is equivalent to a volume constraint:

$$W(\Omega_G) = \frac{300(100 + M_0)d_b}{30000 - 0.265(30 - M_0)d_b} \int_{\Omega_G} d\Omega = \omega * V(\Omega_G) \qquad W(\Omega_G^{Max}) = \omega * V(\Omega_G^{Max}) \tag{12}$$

where V denotes the volume. Hence, the following topological optimization problem

$$\min_{\Omega \in \Pi_{ad}} S_{opt}(\Omega)$$

such that

$$V(\Omega_G) \leq (1-\alpha) V(\Omega_G^{Max}) \quad 0 < \alpha < 1$$

where u is the solution of Equation (11).

Numerical Approach

The numerical approximation is based on the finite element modeling following (Deteix 2008). The finite element discretization of the Galerkin weak form of the mechanical equilibrium (Eq. (11)) and moisture conservation (Eq. (1)) was performed using standard isoparametric and linear interpolation of the unknown displacements u and moisture content, M.

The time discretization of the mass transfer equation was performed by the standard Euler implicit time marching scheme. The predicted values of M, and u depend on position and time. A single system of discrete equation was solved for M at each time step. A user-specified initial time increment of 0.5 s was used. The following time increments were automatically adjusted between 0.1 and 100000 s by the software based on the convergence rate.

Based on the finite element approximation of the displacement and using the SIMP method we produced a discrete optimization problem. The derivatives (or sensitivities) of the system with respect to the volume fraction were then calculated and filtered. The discrete optimization problem was then solved by standard optimizing scheme relying on the filtered derivatives to produce a discrete optimal solution.

Numerical results are presented on the poster, and will be published soon.

Conclusion

The topological optimization strategy proposed is flexible and easily integrated in pre-existing finite element code. The approach can be applied to problem having: more complex geometry, multiple material (EWF, composites, etc), more elaborate physics (boundary conditions, multiple mechanical constraint) or elaborate cost functional.

Even though the application chosen to illustrate this approach is relatively simple and academic in its considerations, it contains, in its formulation and implementation, the principal difficulties that one have to treat when dealing with shape optimization for material under hygromechanical conditions.

As a first approach in shape-topological optimization, the authors consider the results promising. It is clear that the method as to be refined for better integration to practical and industrial application. However, it should not be seen as a mere theoretical exercise.

Environmental considerations and better resources management is becoming paramount in our society, the use of tools such as proposed becomes inevitable.

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