Theoretical Thermal Conductivity Equation for Uniform Density Wood Cells

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Presentation Overview

Background

- Research Goals and Study Objective
- Why model heat transfer in wood
- 2D Finite Element Heat Transfer Models
- Comparisons with other Models

Theoretical Equation Development

- Problem – Interpolation & Use
- Development of the Equations
- Microwave Heating Rates

Applications

- Study of Heat Flux
- Study of Ring Orientation & Density

Background

<u>RESEARCH GOAL</u>:

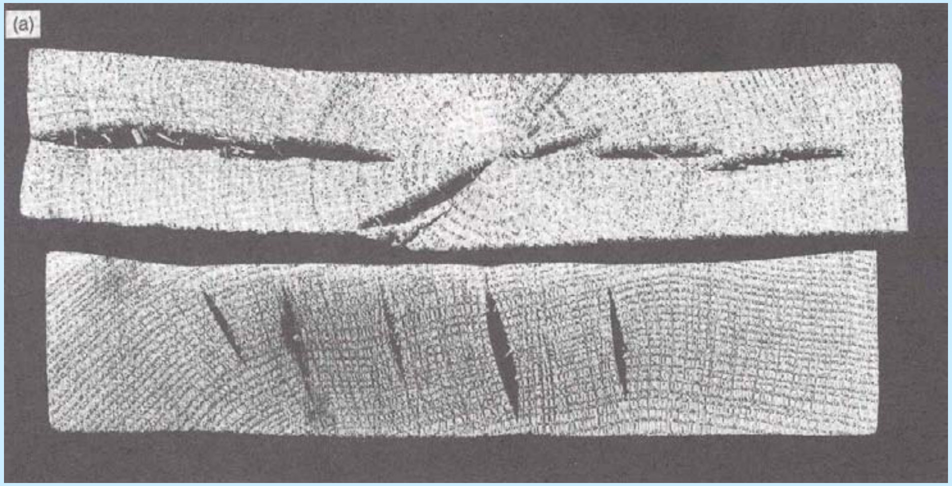
Maintain a healthy and sustainable forest through an understanding of wood and wood processing conditions that can be used to manipulate the properties of wood and wood composites for specific performance characteristics.

<u>STUDY OBJECTIVE</u>:

To understand and develop an easily definable heat transfer coefficient equation that can be used in heat and mass transfer modeling.

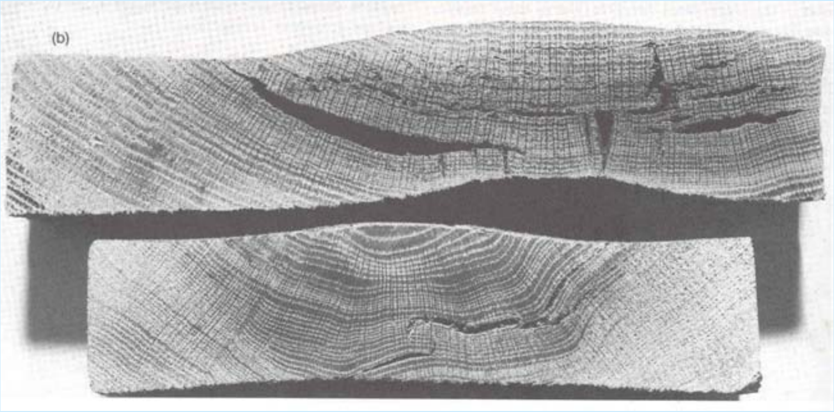
Background

Why model transfer in wood?

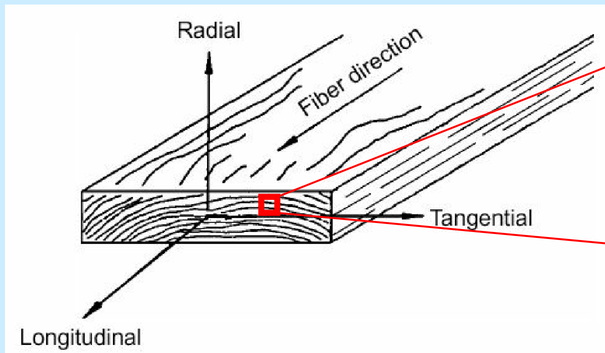


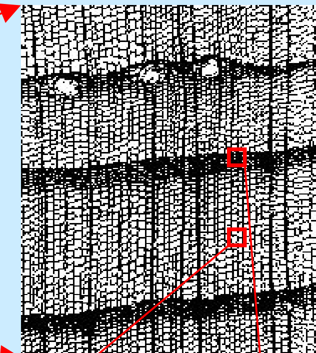
Uncontrolled heating can cause serious defects in the products if not appropriately applied, either with conventional or alternative energy.

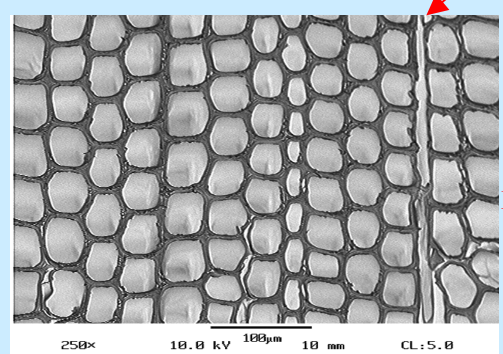
Understanding the flow and generation of heat energy within wood is critical to wood heating control to obtain the desired final product material properties.

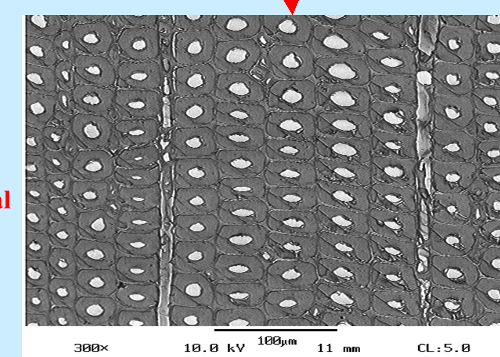


Background
2D Cellular Heat Transfer Model









Radial

Tangential

Softwood is Anisotropic, axial-symmetric, porous material.

Each growth ring consists of two significantly different structures – earlywood and latewood.

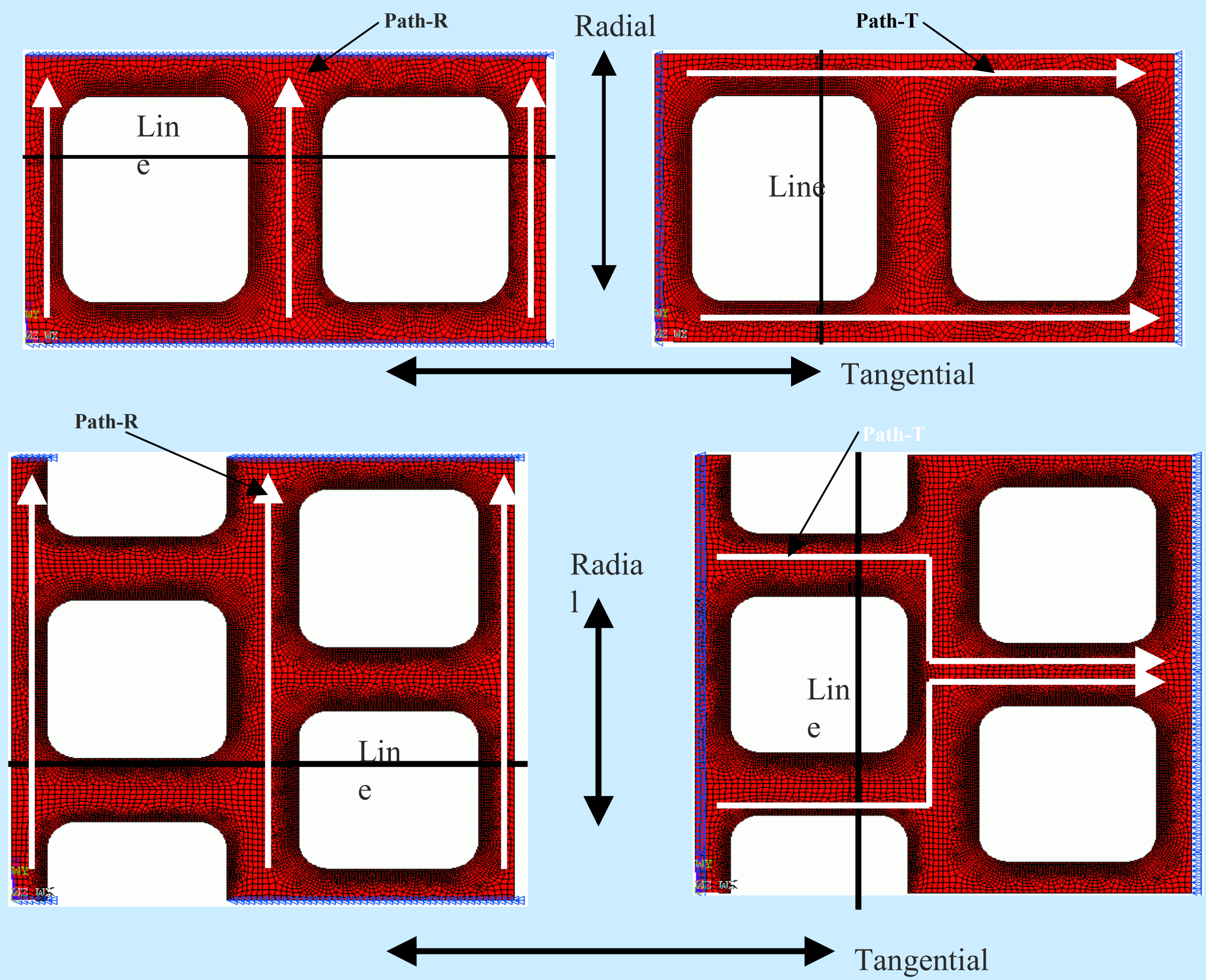
Background

2D Cellular Heat Transfer Model

Input parameters:

@30C

- Cell porosity
- Cell alignment
- Cellwall properties
- Air properties
- Vapor Properties
- Water Properties



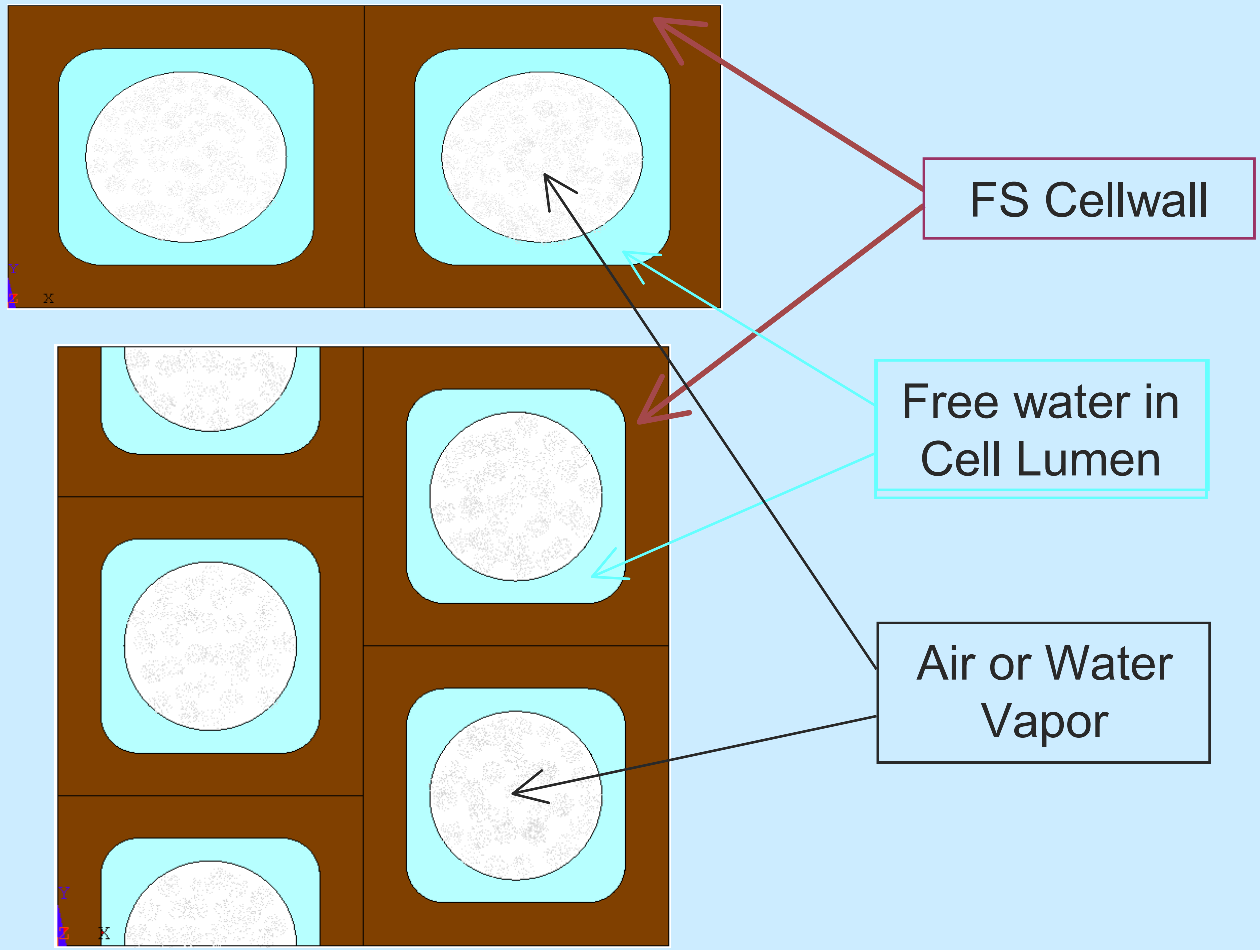
Background
2D Cellular Heat Transfer Model

Cellular Model

Evaluated K_{eff} At:

4 MCs:

- 0% MC Oven Dry
- Fiber Saturation Point (FSP);
- 50/50 water/vapor in the lumen;
- Fully saturated;



Background
2D Cellular Heat Transfer Model

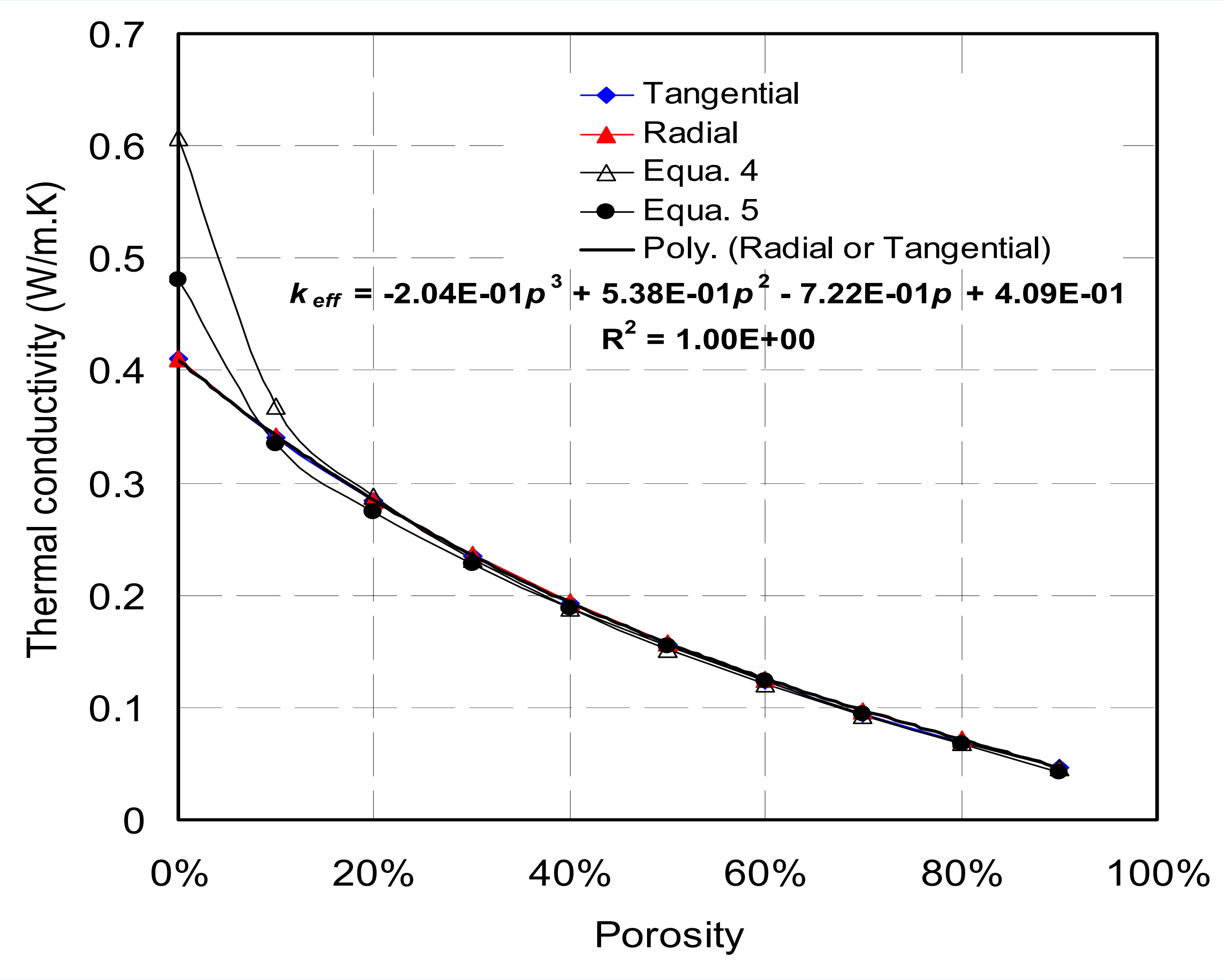
	Symbol	Material Properties in the cellular model Thermal conductivity (W/m.K)	Density (Kg/m^3)	Specific Heat (J/Kg·K)
Cell wall substance (0%MC) [1]	k_{CW}	0.410	1540	1260
Air in the lumen (0%MC) [2]	k_{air}	0.026	1.161	1007
Bound water in cell wall [3]	k_{BW}	0.680	1115	4658
Saturated cell wall (FSP) [4]	k_f [7]	0.489	1415	2256
Water vapor in cell lumen [5]	k_V	0.018	0.734	2278
Free water in cell lumen [6]	k_{FW}	0.610	1003	4176

Note:

1. *Property values for cell wall substance at 0% MC was obtained from Siau's book (1995).*
2. *Air property values for air were obtained from Incropare (1981).*
3. *Density of bound water was obtained from Siau (1995). Thermal conductivity and specific heat of bound water was obtained based on water properties and assumption of the linear relationship with density.*
4. *Property of saturated cell wall was obtained by rule of mixture*
5. *Property values of water vapor was obtained from Ierardi (1999).*
6. *Property values of free water was obtained from Incropera (1981).*
7. *kf is constant when MC is over FSP, but changes with MC below FSP.*

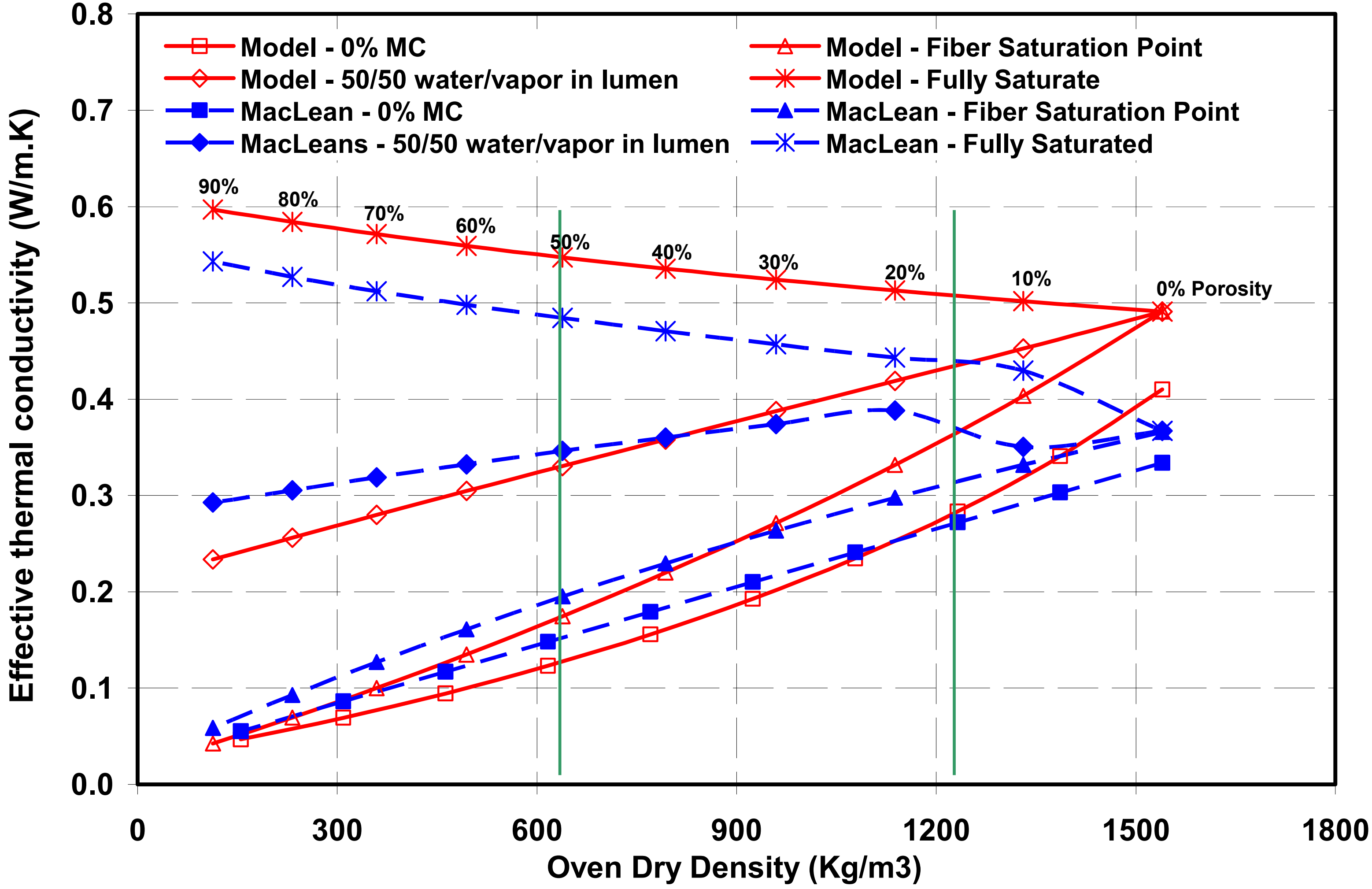
Background
FE Model vs Siau's Model

- Effective thermal conductivity as a function of wood porosity;
- No significant difference between radial and tangential thermal properties;
- Comparing to an electrical circuit model in a wood textbook;



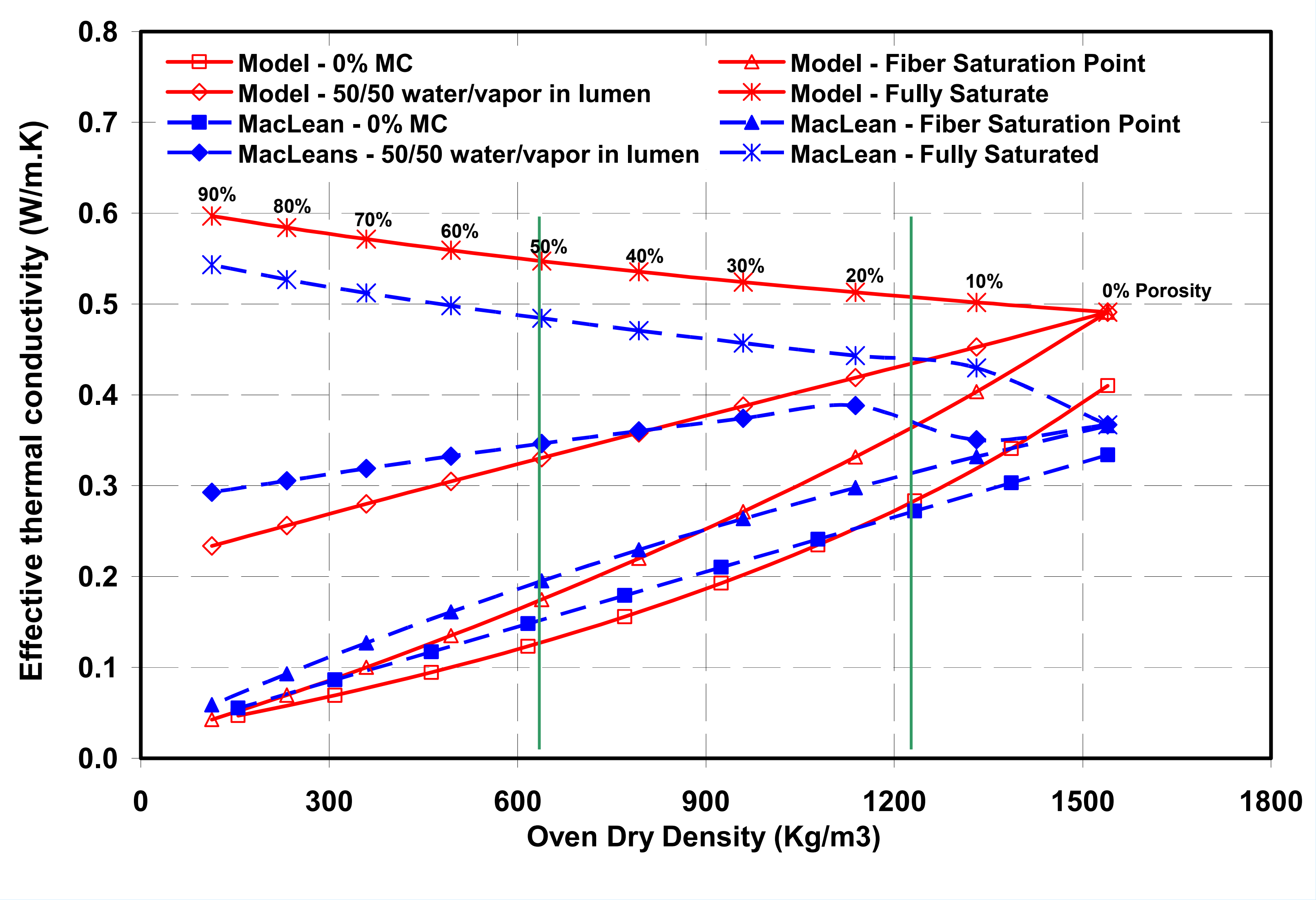
Background

FE Model vs MacLean's Model



Theoretical Equation Development

Problem – Interpolation & Use



Theoretical Equation Development

Development of the Equations

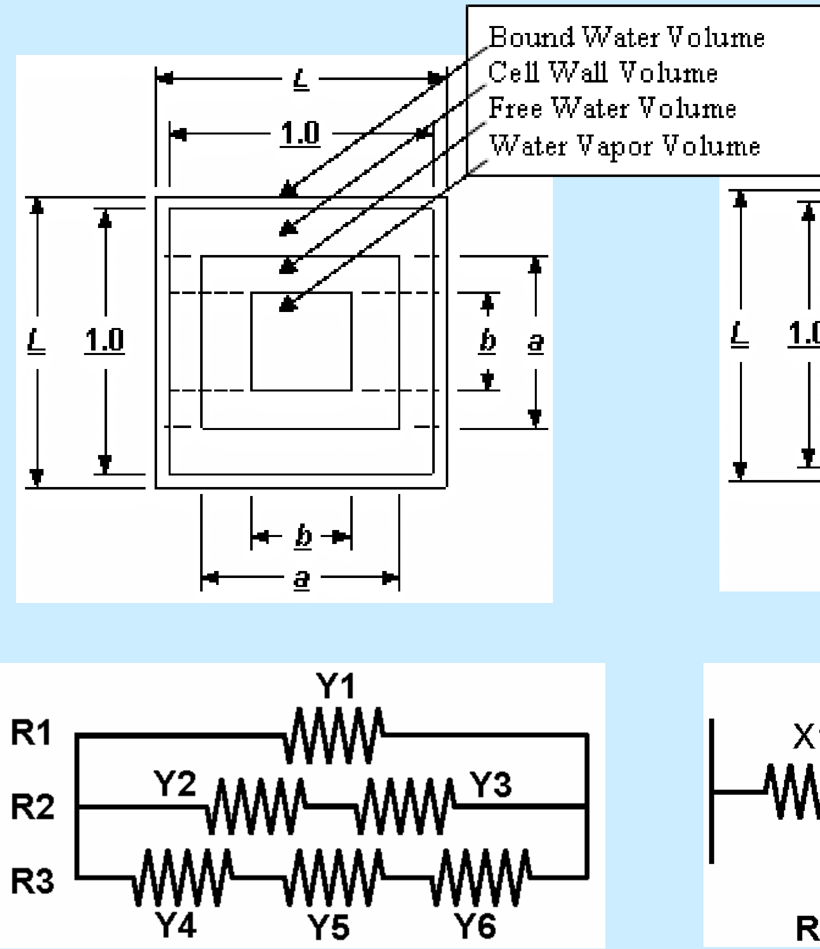
- The Equation needs to be easily useable for future applications;
- Development of one equation that could describe all moisture content and density variations of wood cells;
- Assumes no significant difference between radial and tangential thermal properties;

- 1st Try: Polynomial Equation to fit the data. Nothing worked.;
- 2nd Try: Resistive Equation. ….

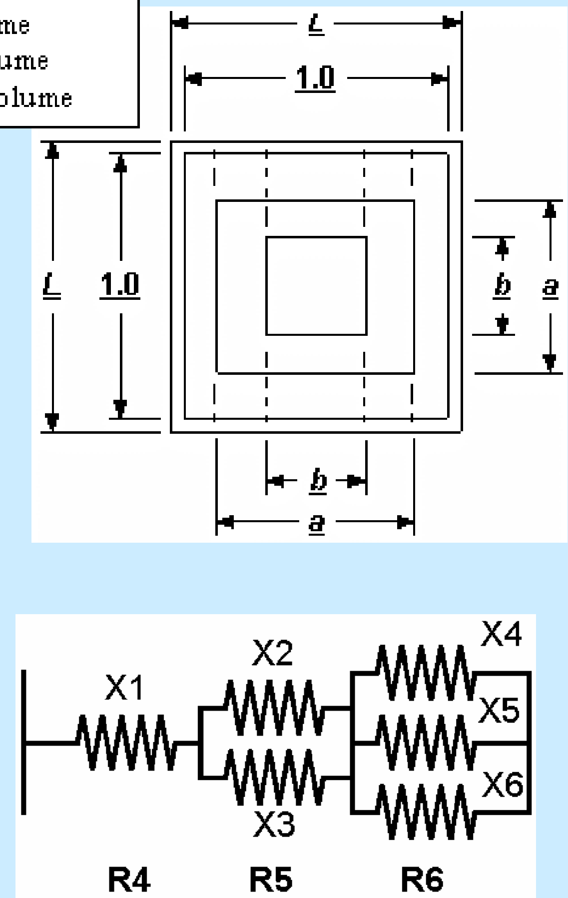
$$K_{eff} = \frac{1}{R_{eff}}$$

Theoretical Equation Development

Resistive Electrical Equations



a. Parallel Circuit

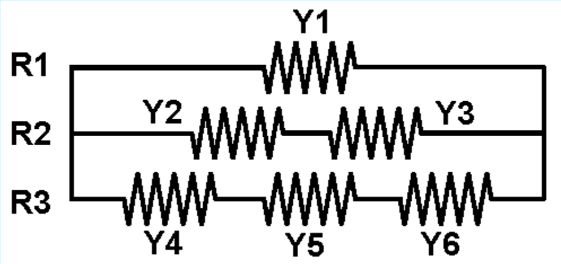


b. Series Circuit

Theoretical Equation Development

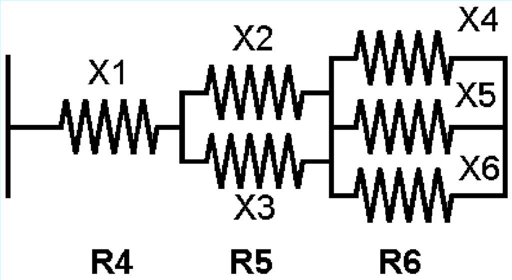
Resistive Electrical Equations

a. Parallel Circuit



$$R_{effp} = \frac{C1C2C3R1R2R3}{C1C2R1R2 + C1C3R1R3 + C2C3R2R3}$$

b. Series Circuit

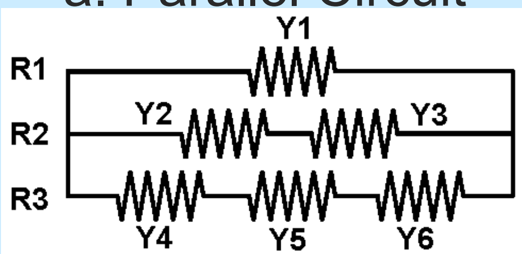


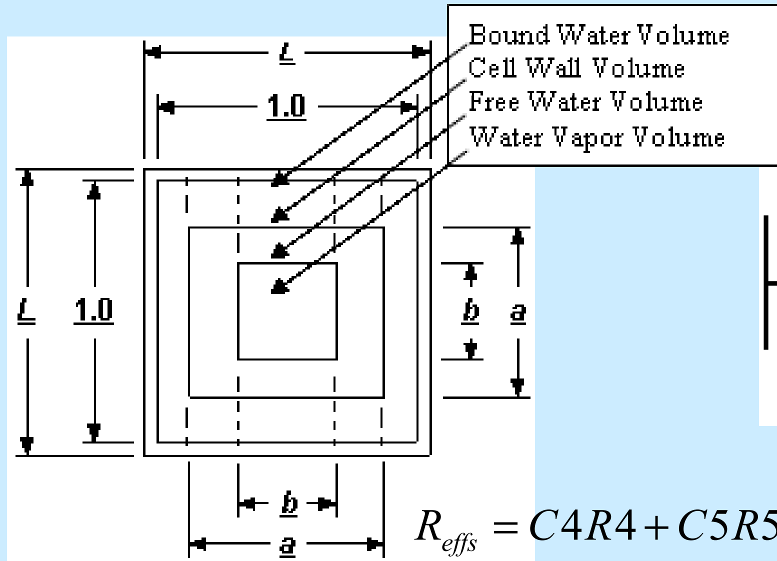
$$R_{effs} = C4R4 + C5R5 + C6R6 = C4X1 + C5\frac{X2X3}{X2 + X3} + C6\frac{X4X5X6}{X4X5 + X4X6 + X5X6}$$

Theoretical Equation Development

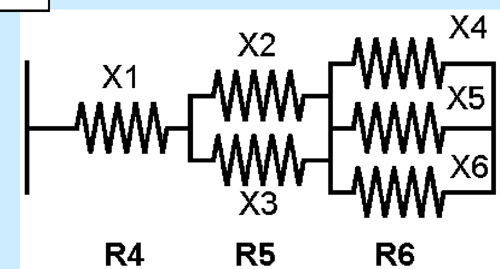
Resistive Electrical Equations

a. Parallel Circuit





b. Series Circuit



$$R_{effp} = \frac{C1C2C3R1R2R3}{C1C2R1R2 + C1C3R1R3 + C2C3R2R3}$$

$$R1 = Y1 = \frac{L}{K_f(L-a)}$$

$$R2 = Y2 + Y3 = \frac{L-a}{K_f(a-b)} + \frac{a}{K_{fw}(a-b)}$$

$$R3 = Y4 + Y5 + Y6 = \frac{L-a}{K_f(b)} + \frac{a-b}{K_{fw}(b)} + \frac{b}{K_v(b)}$$

$$R_{effs} = C4R4 + C5R5 + C6R6$$

$$R_{effs} = C4X1 + C5\frac{X2X3}{X2+X3} + C6\frac{X4X5X6}{X4X5 + X4X6 + X5X6}$$

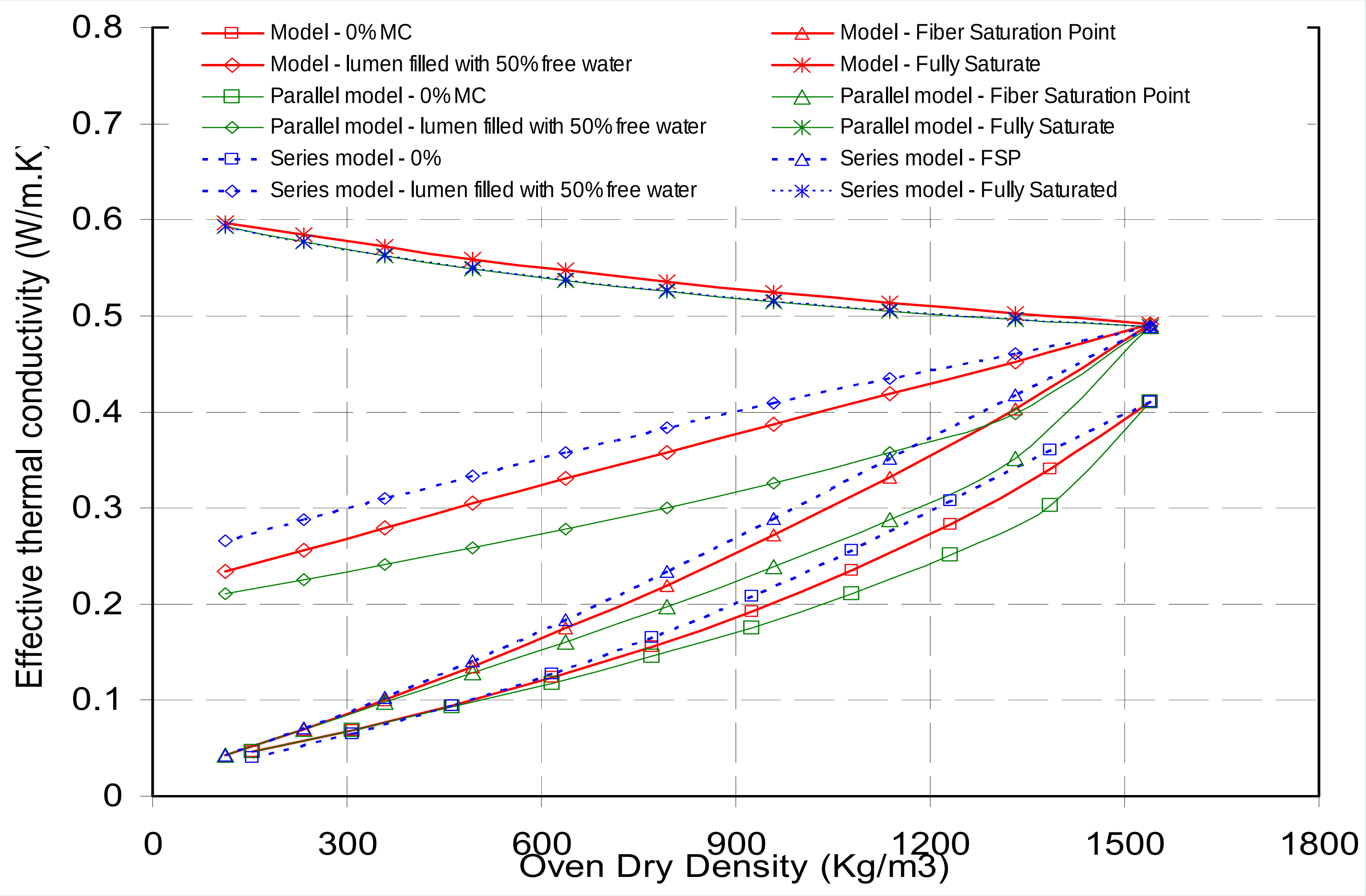
$$X1 = \frac{L-a}{K_f(L)}$$

$$X2 = \frac{a-b}{K_f(L-a)} \qquad X3 = \frac{a-b}{K_{fw}(a)}$$

$$X4 = \frac{b}{K_f(L-a)} \qquad X5 = \frac{b}{K_{fw}(a-b)} \qquad X6 = \frac{b}{K_v(b)}$$

Theoretical Equation Development

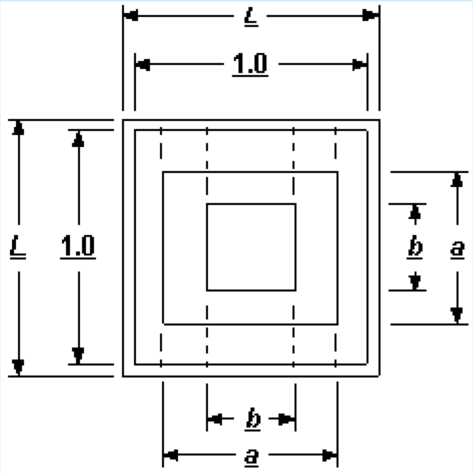
FE vs Parallel vs Series



Theoretical Equation Development

Development of the Equations

The ODD or ρ_{OD}, of the cell is described by density of the cell wall, ρ_{cw}, density of the air, ρ_{air}, and the oven-dry porosity, P_d, in Equ. 21.



$$\rho_{OD} = \rho_{cw}(1 - P_d) + \rho_{air} P_d \qquad (21)$$

P_d is determined by rearranging Equ. 21, Equ. 22.

$$P_d = \frac{\rho_{cw} - \rho_{od}}{\rho_{cw} - \rho_{air}} \qquad (22)$$

Dry porosity can also be described in geometrical terms, Equ. 23.

$$P_d = \frac{a^2}{1^2} \qquad (23)$$

Theoretical Equation Development

Development of the Equations

With the addition of moisture into the cell, the volume percent of bound water, $V\%_{bw}$, in the cell wall needs to be described. By definition, fiber moisture content MC_f is calculated by dividing bound water weight by the oven-dry cell weight, which is the cellulose material weight, when assuming UNIT volume for the wood cell. The MC in the fiber cell wall (MC_f) can be calculated using Equ. 24.

$$MC_f = \frac{V\%_{bw}\rho_{cw}}{\rho_{cw}(1 - V\%_{bw})} \qquad (24)$$

By rearranging Equ. 24, $V\%_{bw}$ can be determined for MC < 0.3. For MC > 0.3 $V\%_{bw}$ is a constant at 0.293. For MC from 0 to 0.3 then $MC_f \cong$ MC. For MC > 0.3 $MC_f = 0.3$

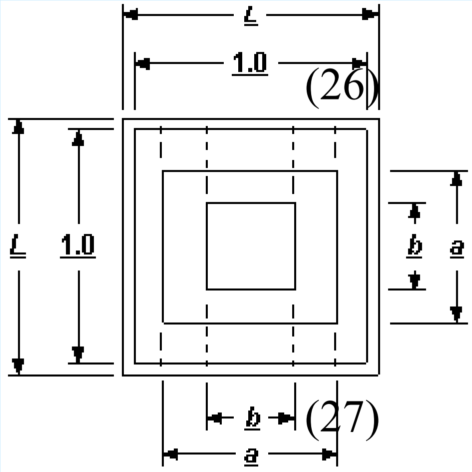
$$V\%_{bw} \cong \frac{MC_f\rho_{cw}}{MC_f\rho_{cw} + \rho_{bw}} \qquad (25)$$

Theoretical Equation Development

Development of the Equations

With the addition of moisture into the cell, wet porosity, P_w, is introduced and can be described using Equ. 26.

$$P_w = \frac{(1 - V\%_{bw})P_d}{1 - V\%_{bw} P_d} \qquad (26)$$



Wet porosity can also be described in geometrical terms, Equ. 27.

$$P_w = \frac{a^2}{L^2} \qquad (27)$$

The outer dimension, L, and area of the wet cell, L^2, changes with increasing moisture up to the FSP and can be described using Equ. 28.

$$L^2 = \frac{\rho_{bw} + MC_f \rho_{cw}}{\rho_{bw} + MC_f P_w \rho_{cw}} \qquad (28)$$

Theoretical Equation Development

Development of the Equations

The specific equation for MC at all moisture conditions is described in Equ. 29.

$$MC = \frac{(\rho_{cw} V\%_{bw})(1-P_w) + \dfrac{\rho_v b^2 P_w}{a^2} + \rho_{fw}(1-\dfrac{b^2}{a^2})P_w}{(\rho_{cw}(1-V\%_{bw}))(1-P_w)} \qquad (29)$$

The location and description of the free water is around the inner lumen that extents into the middle of the lumen having a dimension *b*. For MC < 0.3 there is assumed no free water then *b* = 0, Fig. 4. For MC > 0.3 it is assumed the cell wall is saturated and the remaining water goes into the lumen as free water which can be determined by rearranging Equ. 28 and solving for *b*, Equ. 30.

$$b = \sqrt{\frac{a^2\{(1-P_w)[V\%bw\rho_{bw} - (1-V\%_{bw})MC\rho_{cw}] + P_w\rho_{fw}\}}{P_w(\rho_{fw} - \rho_v)}} \qquad (30)$$

Theoretical Equation Development

Development of the Equations

Thermal conductivity for the fiber cell wall material, K_f, needs to also be determined before calculating the effective thermal conductivity. Using rules of mixtures for MC < 0.3 then, K_f can be determine by Equ. 31.

$$K_f = K_{cw}(1 - V\%_{bw}) + K_{bw} V\%_{bw} \qquad (31)$$

For MC > 0.3 the fiber cell wall is fully saturated with water and is assumed does not change. Using the rule of mixtures for $MC_f = 0.3$ then the fiber cell wall thermal conductivity is a constant at $K_f = 0.4891$, Table 1.

Theoretical Equation Development

Development of the Equations

The effective thermal conductivity, K_{eff}, for the entire range of moisture contents and densities can be determined by only knowing the ODD and MC conditions for the sample by substituting Equ.s 15-20 are into Equ. 14 and simplifying the equation as Equ. 32.

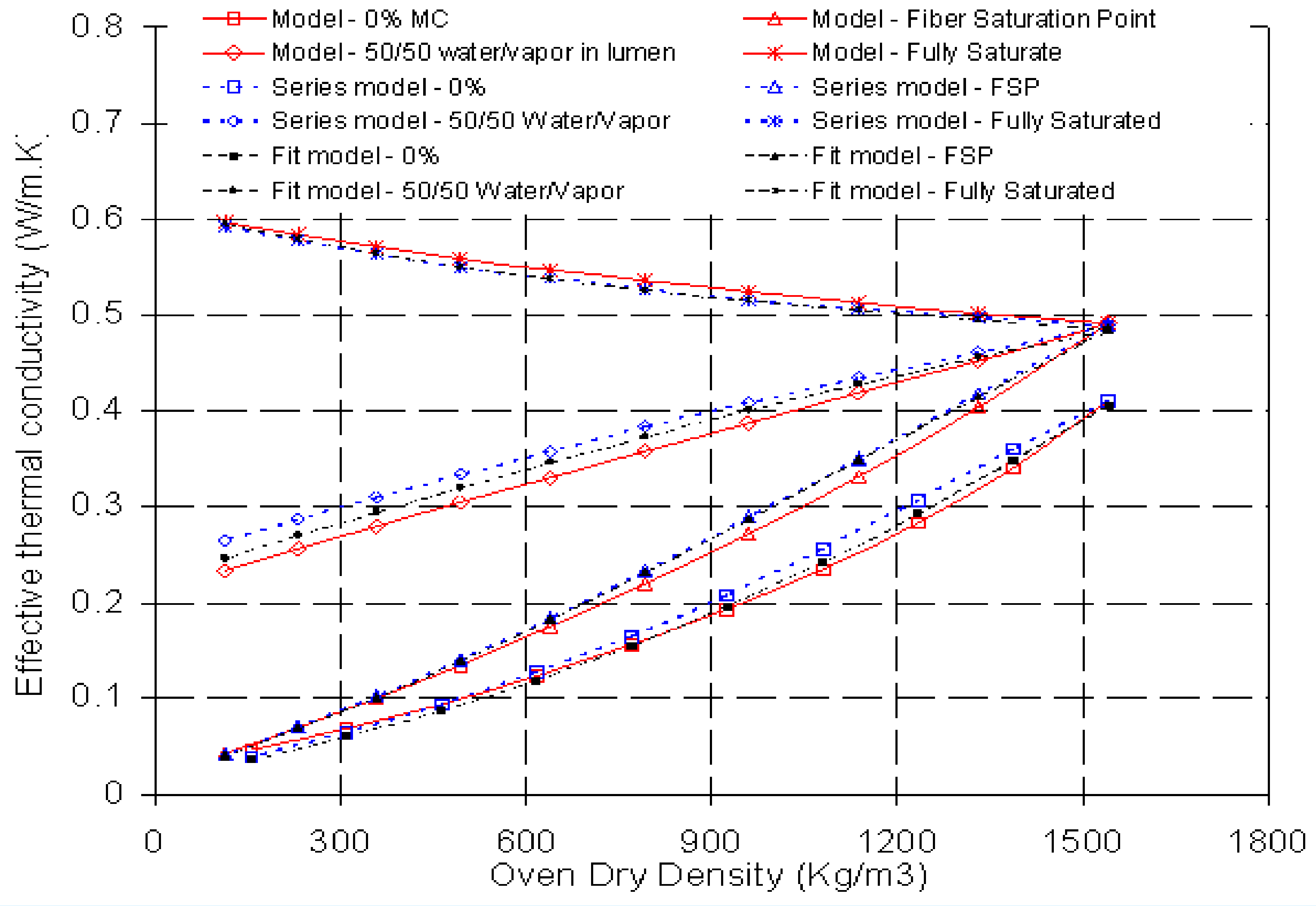
$$K_{eff} = \frac{(L-a)K_f}{(a-L)((\frac{a}{L}-1)C4+K_f(\frac{(a-b)C5}{(a-L)K_f-aK_{fw}}+\frac{bC6}{(a-L)K_f+(b-a)K_{fw}-bK_v}))} \quad (32)$$

By solving for and substituting the appropriate values for variables *a, b, L,* K_f and substituting the appropriate *C4, C5*, and *C6* parameters in Table 2, the entire range of thermal conductivities can be determined.

Similarly, the equation can be used to determine estimated thermal conductivity values where FSP is other than 30%. By changing the volume percent of bound water, $V\%_{bw}$, (Equ. 25) and recalculating the values.

Theoretical Equation Development

Equation with Constants

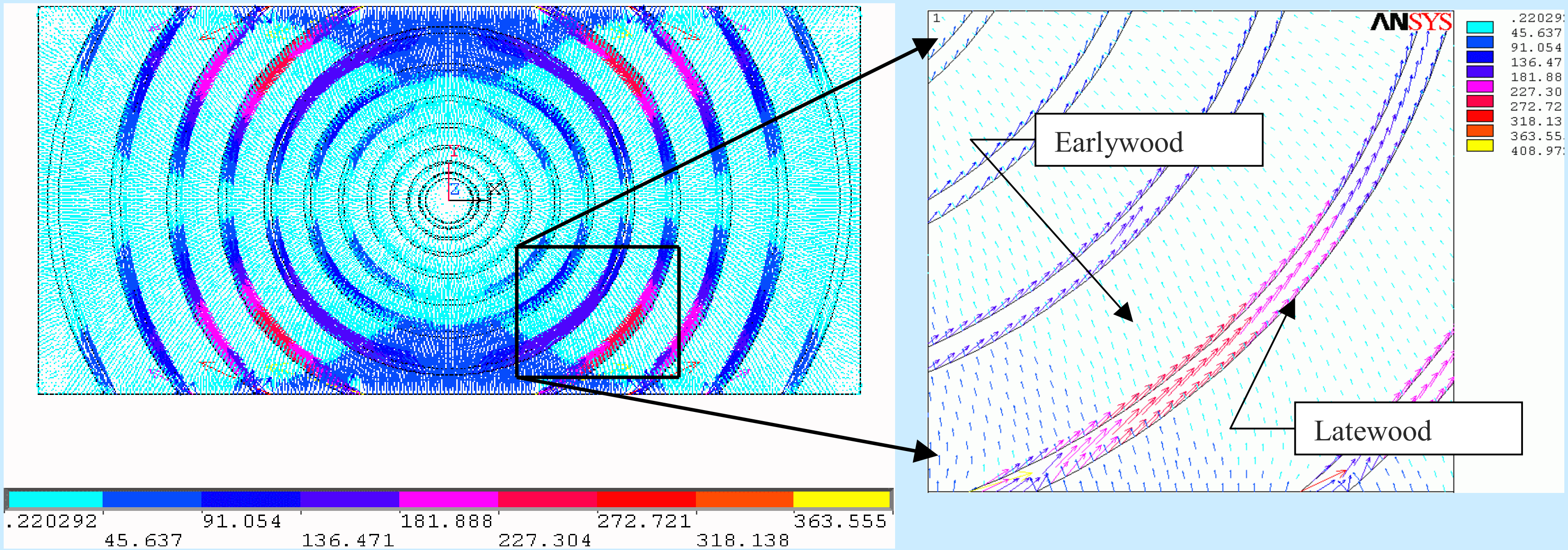


Application

Study of Heat Flux

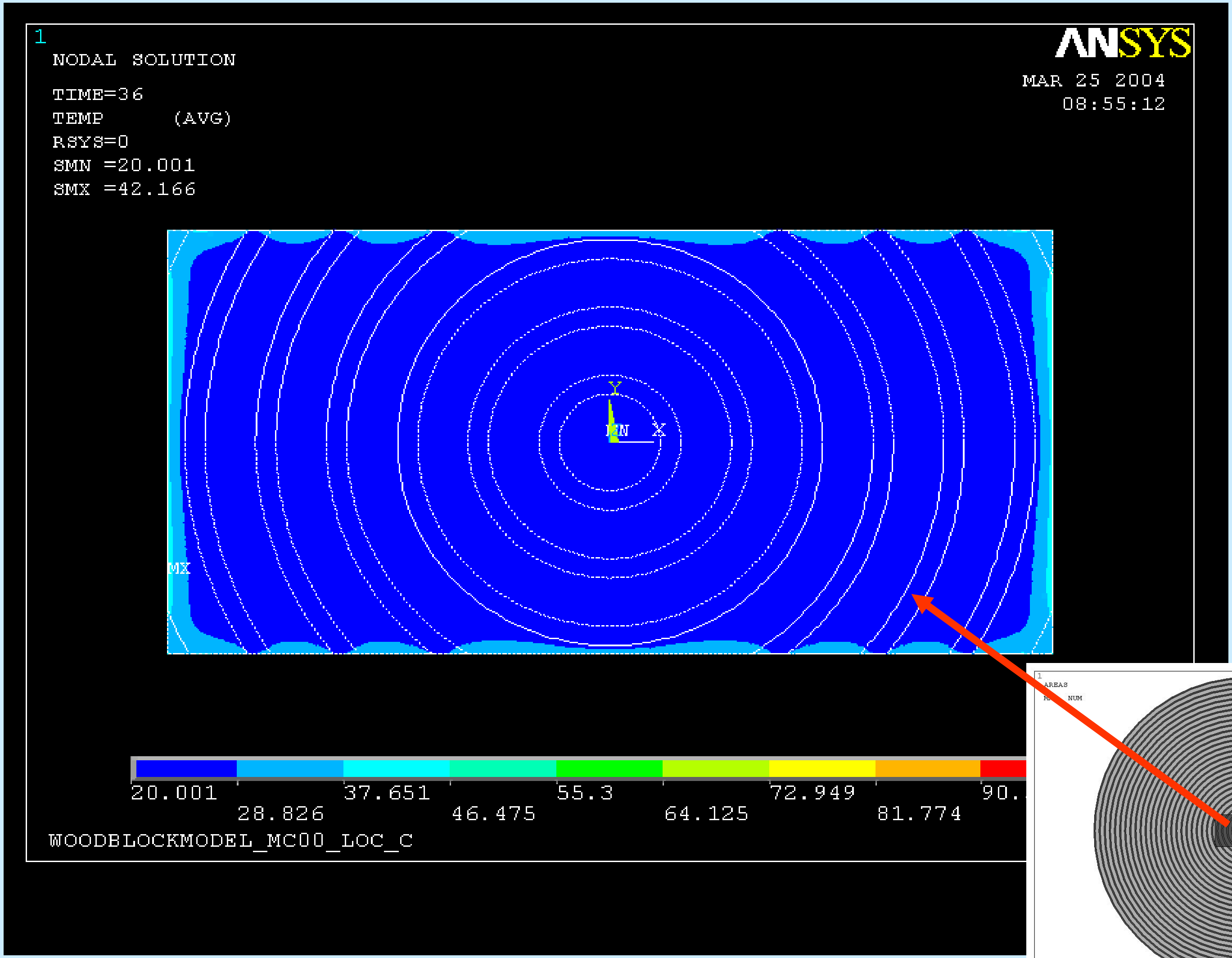
Heat flow in a wood board:

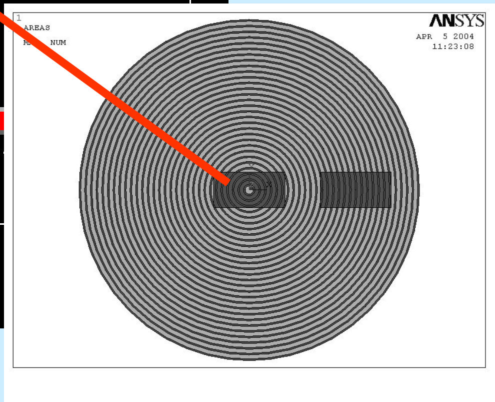
- Significant heat flux follows the latewood rings;
- Higher energy is shown being transferred through the latewood rings, then across into the earlywood from one latewood ring to the next toward the center of the board



Application

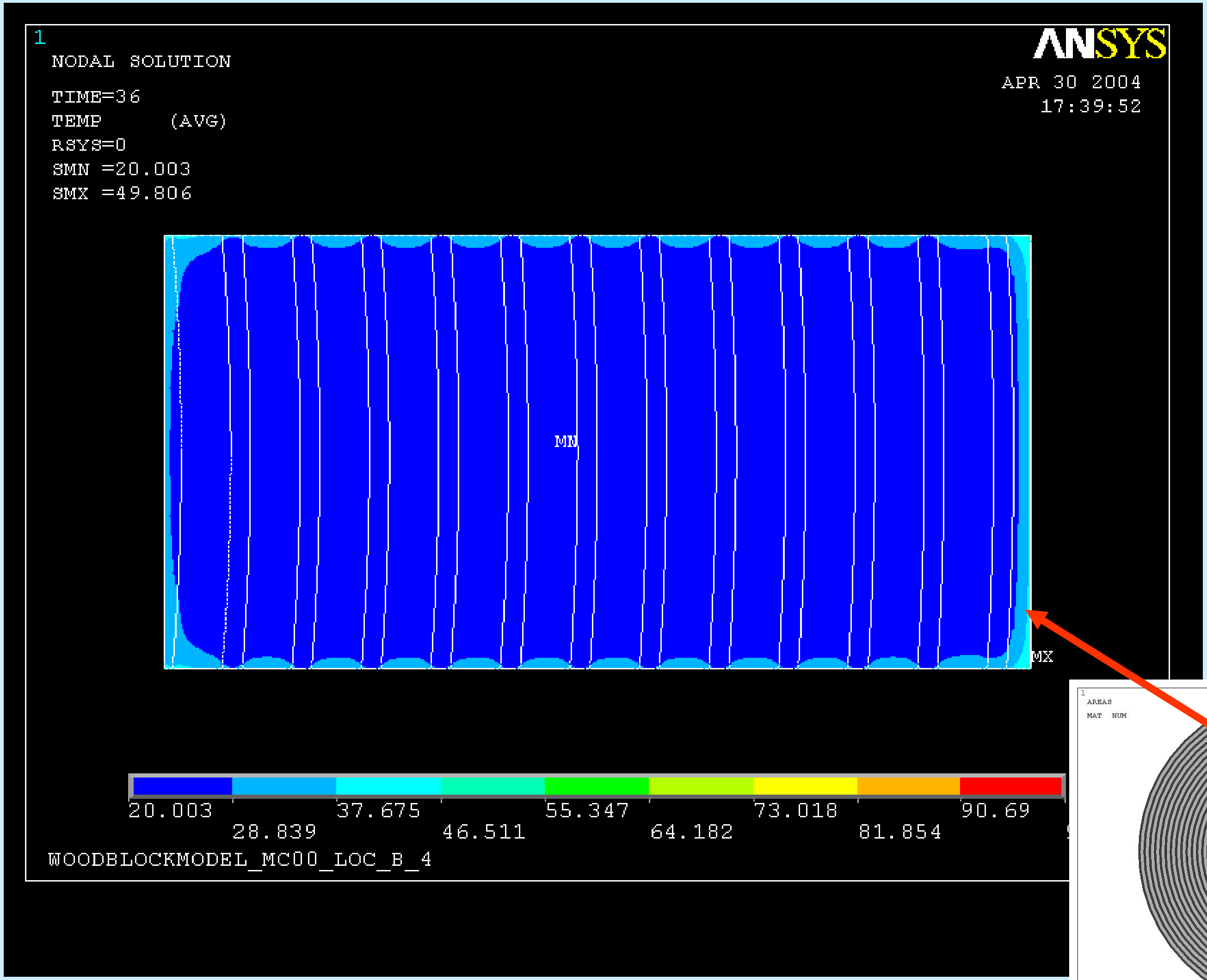
Study of Temperature

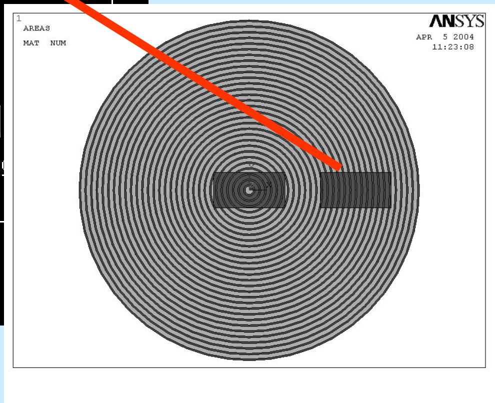




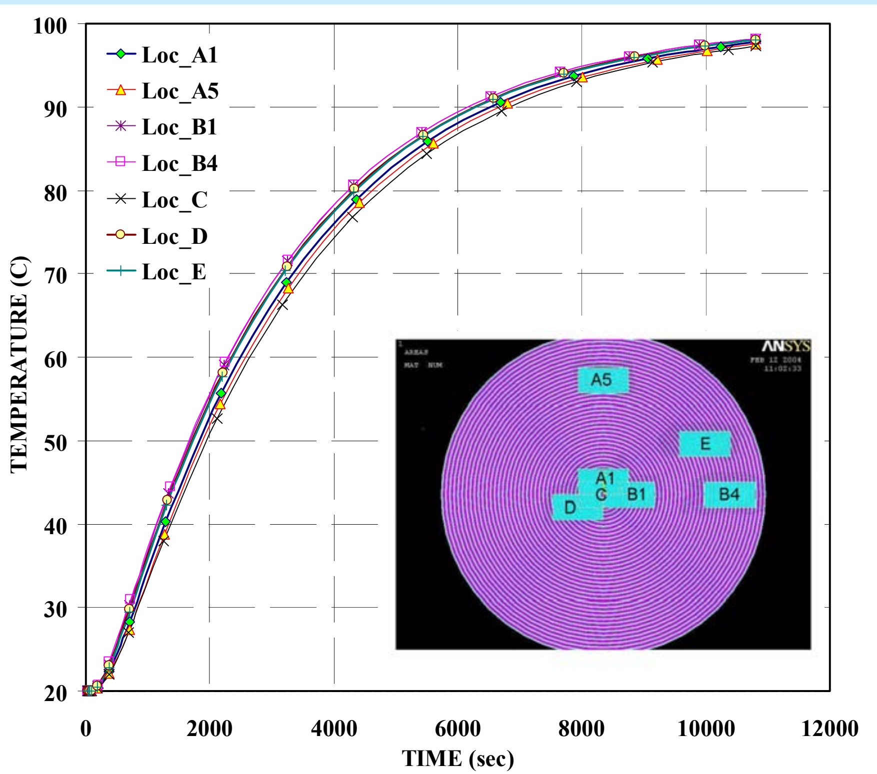
Application

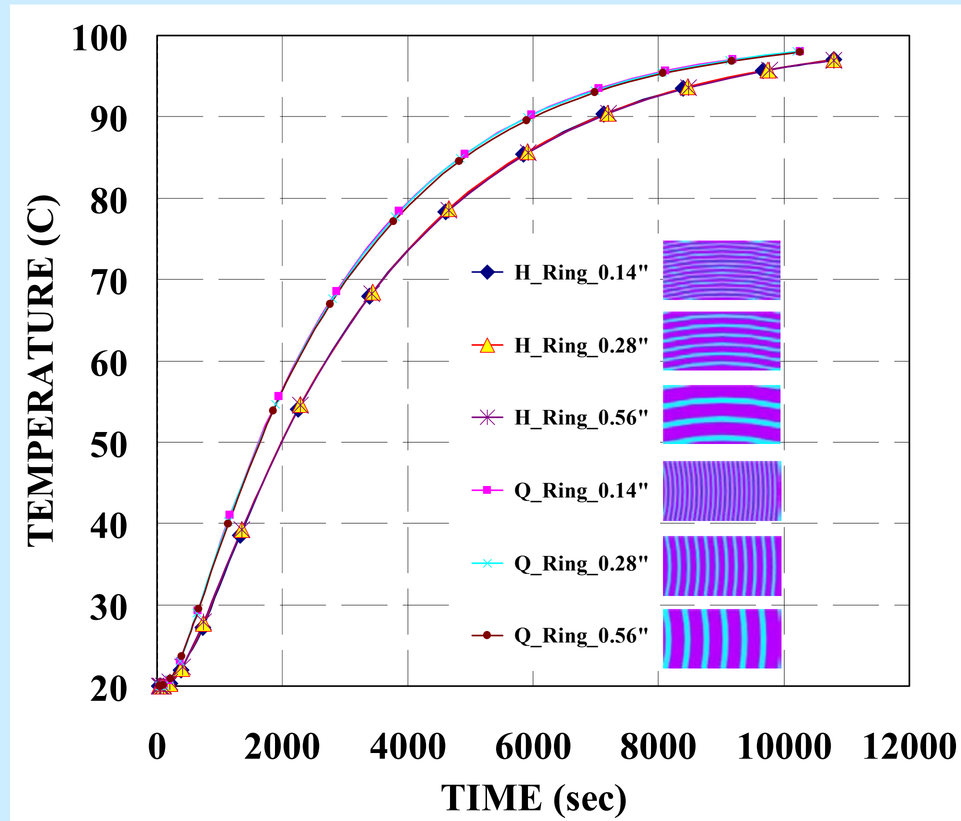
Study of Temperature





Application
Study of Ring Orientation & Density





In summary: I would like to say
Thank You

Any Questions